$f_2(1270)$, $a_1(1260)$ and $f_0(1370)$ as dynamically reconstructed quark-antiquark states*

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Abstract

It is explained why the interpretation of the resonances $f_2(1270)$, $a_1(1260)$ and $f_0(1370)$ as quark-antiquark states is legitimate. The result of the quark model and of recently performed Bethe-Salpeter studies are not (necessarily) in conflict and can be understood as two different approaches toward the description of the same quark-antiquark resonances.

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1 Introduction

The interpretation of hadronic resonances is an important subject of low-energy hadron physics. Can all resonances be described in terms of the building blocks of QCD, quarks and gluons? Are there, on the contrary, 'dynamically generated' resonances, which emerge as molecular bound states upon interactions of other, more fundamental hadrons?

In this work we shall concentrate on some particular mesons in the mass range 1-1.5 GeV: $f_2(1270)$, $a_1(1260)$ and $f_0(1370)$. These states were investigated both in the old-fashioned quark model [1] and in more recent studies [2]. In the quark model they are interpreted as quark-antiquark pairs: $f_2(1270) \equiv \bar{n}n \equiv \sqrt{1/2}(\bar{u}u + \bar{d}d)$ with quantum numbers $J^{PC} = 2^{++}$, $a_1^0(1260) \equiv \sqrt{1/2}(\bar{u}u - \bar{d}d)$ with quantum numbers $J^{PC} = 1^{++}$ (similarly for the other charged states), $f_0(1370) \equiv \bar{n}n \equiv \sqrt{1/2}(\bar{u}u + \bar{d}d)$ with quantum numbers $J^{PC} = 0^{++}$. Indeed, the quarkonium assignment describes very well the masses and also the decays properties for the whole tensor meson nonet $J^{PC} = 2^{++}$ [3], to which $f_2(1270)$ belongs. It also functions well for the axial-vector meson nonet $J^{PC} = 1^{++}$, to which $a_1(1260)$ belongs. The scalar mesons, such as $f_0(1370)$, are as usual controversial objects in QCD, however the interpretation of $f_0(1370)$ as predominantly quarkonium is in agreement with the present results, see Refs. [4, 5] and refs. therein for the presentation of various mixing patterns.

In the works of Refs. [2] the very same resonances $f_2(1270)$, $a_1(1260)$ and $f_0(1370)$ have been obtained in -at first sight- utterly different light: the resonances $f_2(1270)$ and $f_0(1370)$ are interpreted as $\rho\rho$ molecular states, and $a_1(1260)$ is interpreted as $\rho\pi$ molecular state. These works are based on the Bethe-Salpeter (BS) equation applied to mesonic low-energy chiral Lagrangians describing $\rho\rho$ and $\rho\pi$ interactions.

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The two descriptions seem mutually exclusive: a mesonic molecular state is a different object than a quark-antiquark bound state. This is surely true, but a closer look at the problem is necessary. Namely, to which extent can one conclude that these resonances are hadronic molecular states? Surely, the BS equation represents a well-defined field theoretical framework to describe bound states. However, the BS approach is used in the context of low-energy hadronic theories as a unitarization method: the BS-resummation scheme is applied to a low-energy effective Lagrangians with a limited range of validity. The masses of the poles of BS amplitudes lie above the validity of the corresponding low-energy hadronic theory. This is indeed a subtle point that requires a careful study, which we will present in this paper along the line of Ref. [6].

It is useful to discuss the problem with the help of two examples:

(a) Positronium states in QED: the QED Lagrangian contains two fields, the photon and the electron. Further composite states with a mass of about $2m_e$ emerge upon electron-positron interactions. These states are the positronia, i.e. bound states of electron and positron due to photon exchange. Positronia are genuine dynamically generated states of molecular type which do not appear in the original QED Lagrangian.

(b) Fermi-Lagrangian and the nature of the W meson: in the standard model (SM) the fields W, electron and neutrino are elementary. The mass of the weak W boson is however very large (80 GeV). When integrating out from the SM the W-field, the Fermi Lagrangian for e-ν interaction emerges. If one applies unitarization techniques by resumming e-ν loops to the Fermi Lagrangian, the existence of the W meson can be inferred. However, this does not mean that the W meson is a 'dynamically generated' bound state of an electron and a neutrino. Indeed, the W meson is exactly as elementary as e and ν. One can rather say that the W meson can be 'reconstructed' by unitarizing the low-energy Fermi Lagrangian.

The question concerning the resonances $f_2(1270)$, $a_1(1260)$, and $f_0(1370)$ can be summarized as follows: are they analogous to the case (a) or the case (b)? In this work we argue that they are analogous to the case (b). This means that these resonances are not hadronic molecular bound states, but rather standard quarkonia. They are obtained upon unitarizations of low-energy hadronic Lagrangians, just as the W meson can be obtained from the low-energy Fermi Lagrangian.

2 Dynamical reconstruction of quark-antiquark states

2.1 A short survey of low-energy theories of QCD

The QCD Lagrangian $\mathcal{L}_{QCD}$ contains quarks and gluons, which due to confinement are not the relevant degrees of freedom at low energy. The proper degrees of freedom are colorless hadron states. The effective Lagrangian describing these hadrons up to (some) maximal energy $E_{\max}$ is denoted as $\mathcal{L}_{had}(E_{\max}, N_c)$.

We briefly describe four important particular cases of $\mathcal{L}_{had}(E_{\max}, N_c)$.

(i) The case $E_{\max} \simeq 2$ GeV is interesting from a phenomenological point of view, because all the low-lying nonets ((pseudo)scalars and (axial)vectors) lie below this energy. Unfortunately, $\mathcal{L}_{had}(E_{\max} \simeq 2GeV, N_c = 3)$ is unknown. It is in fact not possible to derive it from $\mathcal{L}_{QCD}$. (For recent attempts to describe -part of- $\mathcal{L}_{had}(E_{\max} \simeq 2\text{ GeV}, N_c = 3)$ including (pseudo)scalar and (axial)vector mesons see Ref. [5]).

(ii) When setting $E_{\max} = E_{\chi PT} \simeq 300$ MeV, the Lagrangian $\mathcal{L}_{had}(E_{\max} \simeq 300\text{ MeV}, N_c = 3)$ contains only the 3 light pions:

$$\mathcal{L}_{\chi PT} = \mathcal{L}_{had}(E_{\max} = E_{\chi PT}, N_c = 3) = \sum_{k=1}^{3} \left[\frac{1}{2} \left(\partial_\mu \pi_k\right)^2 - \frac{1}{2} M_\pi^2 \pi_k^2 \right] + \mathcal{L}_{int}^{\pi} \ , \tag{1}$$

whereas $\mathcal{L}_{int}^{\pi}$ describes the interaction term. This is indeed the Lagrangian of chiral perturbation theory [7], whose terms, but not the related coupling constants (known as low-energy coupling constants, LECs), can be determined by considerations based on chiral symmetry. Note, if $\mathcal{L}_{had}(E_{\max} \simeq$

$2GeV, N_c = 3)$ were known it would be possible, upon integrating out all the heavier fields, to determine exactly $\mathcal{L}_{\chi PT}$: both the operators and the LECs would be calculable. In the next subsection a toy model where this operation is possible is shown.

(iii) If, instead, we chose $E_{\max} \simeq 1$ GeV we obtain the effective Lagrangian

$$\mathcal{L}_{had}(E_{\max} \simeq 1 \text{ GeV}, N_c = 3) = \mathcal{L}_{\chi PT+VM} \ , \tag{2}$$

i.e. a Lagrangian which describes the pseudoscalar mesons, the vector mesons and their interactions [8]. It is out of this Lagrangian that the resonances $f_2(1270)$, $a_1(1260)$, $f_0(1370)$ where obtained in Ref. [2] upon unitarization based on the BS-equation.

(iv) There is one theoretical limit in which $\mathcal{L}_{had}(E_{\max}, N_c)$ can be determined: the large-N_c limit. In fact, for $N_c >> 1$ the theory $\mathcal{L}_{had}(E_{\max}, N_c >> 1)$ contains only free quarkonia and glueballs with a mass below the maximal energy $E_{\max}$:

$$\mathcal{L}_{had}(E_{\max}, N_c >> 1) = \sum_{k=1}^{N_{\overline{q}q}} \left[\frac{1}{2} \left(\partial_\mu \phi_k\right)^2 - \frac{1}{2} M_{\overline{q}q,k}^2 \phi_k^2 \right] + \sum_{h=1}^{N_{gg}} \left[\frac{1}{2} \left(\partial_\mu G_h\right)^2 - \frac{1}{2} M_{G,h}^2 G_h^2 \right] \ . \tag{3}$$

2.2 A toy-model and its analogy with the hadronic world

In order to explain the issue it is useful to introduce a simple toy-model with the scalar fields φ and S [6, 9]:

$$\mathcal{L}_{\text{toy}}(E_{\max}, N_c) = \frac{1}{2} \left(\partial_\mu \varphi\right)^2 - \frac{1}{2} m^2 \varphi^2 + \frac{1}{2} \left(\partial_\mu S\right)^2 - \frac{1}{2} M_0^2 S^2 + g(N_c) S \varphi^2 - \frac{g(N_c)^2}{2M_0^2} \varphi^4 \ , \tag{4}$$

where the large-N_c dependence is expressed via the scaling $g(N_c) = g_0 \sqrt{3/N_c}$. The two masses are large-N_c independent and the decay width $S \to 2\varphi$

$$\Gamma_{S \to \varphi\varphi} = \frac{\sqrt{\frac{M_0^2}{4} - m^2}}{8\pi M_0^2} \left[\sqrt{2} g(N_c)\right]^2 \tag{5}$$

scales as $1/N_c$, exactly as if φ and S were quark-antiquark states. (We assume that $M_0 > 2m$ and that the validity of the Lagrangian $\mathcal{L}_{\text{toy}}$ is such that $E_{\max} >> M_0$).

Let us now turn to the determination of an effective low-energy model of this simplified system. We integrate out S and obtain a low-energy Lagrangian $\mathcal{L}_{\text{le}}$ valid in the interval $E_{\text{le}} \lesssim 2m < M_0$ and depending *only* on the field φ:

$$\mathcal{L}_{\text{le}}(E_{\text{le}}, N_c) = \frac{1}{2} \left(\partial_\mu \varphi\right)^2 - \frac{1}{2} m^2 \varphi^2 + V, \ V = \sum_{k=1}^{\infty} V^{(k)} \ , \tag{6}$$

$$V^{(k)} = L^{(k)} \varphi^2 \left(-\Box\right)^k \varphi^2, \ L^{(k)} = \frac{g(N_c)^2}{2M_0^{2+2k}} \ . \tag{7}$$

It is easy to establish an analogy of the toy-model with the real hadronic world, see Table 1. $\mathcal{L}_{\text{toy}}(E_{\max}, N_c)$ corresponds to the (unknown) hadronic Lagrangian $\mathcal{L}_{had}(E_{\max} \simeq 2 \text{ GeV}, N_c)$, while the low-energy Lagrangian $\mathcal{L}_{\text{le}}(E_{\text{le}}, N_c)$ corresponds to a low-energy hadronic Lagrangian, such as $\mathcal{L}_{\chi PT}$ or $\mathcal{L}_{\chi PT+VM}$.

Table 1: Analogy

Toy-Model	Hadronic world
$\mathcal{L}_{\text{toy}}(E_{\max}, N_c)$	$\mathcal{L}_{had}(E_{\max} \simeq 2 \text{ GeV}, N_c)$
$\mathcal{L}_{\text{le}}(E_{\text{le}}, N_c)$	$\mathcal{L}_{\chi PT}$ or $\mathcal{L}_{\chi PT+VM}$

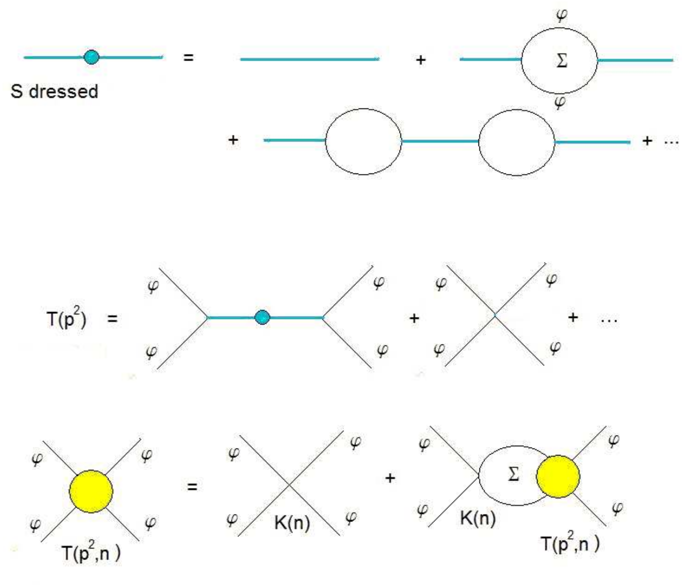

Figure 1: First row: Dressing of S-field through loops of φ-fields. Second row: Pictorial representation of the 1-loop resummed T-matrix $T(p^2)$ of Eq. (8). Third row: Pictorial representation of the BS-approximation $T_{BS}(p^2, n)$ of Eq. (9).

There is however a crucial difference: while in the toy-model the knowledge of the 'full Lagrangian' $\mathcal{L}_{\rm toy}(E_{\rm max}, N_c)$ allows to determine the low-energy counterpart $\mathcal{L}_{\rm le}(E_{\rm le}, N_c)$ precisely up to an arbitrary order n (see Eq. (6)), the Lagrangians $\mathcal{L}_{\chi PT}$ and $\mathcal{L}_{\chi PT+VM}$ are only partially known. The terms are determined via symmetry considerations, but the corresponding coupling constants (LECs), which are analogous to the L_k in Eq. (7), cannot be calculated: they must be obtained via comparison with experiments. This fact represents also a practical limit of low-energy effective theories: although it is in principle possible up to work at each n, the technical difficulty due to the fast increasing number of terms and the large number of unknown related LECs render the calculations doable only up to the third order.

2.3 The concept of dynamical reconstruction

In the framework of the toy-model, the T-matrix for $\varphi\varphi$ scattering in the s-channel can be calculated from the Lagrangian $\mathcal{L}_{\rm toy}(E_{\rm max}, N_c)$ (at 1-loop, see the first and the second rows of Fig.1):

$$T(p^2) = \frac{1}{-K^{-1} + \Sigma_\Lambda(p^2)},\ K = \frac{(\sqrt{2}g)^2}{M_0^2 - p^2} - \frac{(\sqrt{2}g)^2}{M_0^2}\ , \tag{8}$$

where $\Sigma_\Lambda(p^2)$ is the loop function, which depends on a cutoff Λ, see Ref. [9] for details. This is for our purposes the 'exact' T-matrix of the problem [1].

Let us now consider the low-energy Lagrangian $\mathcal{L}_{\rm le}(E_{\rm le}, N_c)$, in which the potential V is approximated at the order n: $V(n) = \sum_{k=1}^{n} V^{(k)}$. We can apply a BS-study to this system, see the third row

[1] Clearly this form of the T-matrix is valid in the 1-loop approximation. Even this simple QFT is not exactly solvable. Neverhteless, the resummed 1-loop expression is regarded as 'exact' in comparison to the approximated BS-form derived later.

of Fig.1, obtaining upon resummation the following approximated form of the T-matrix:

$$T_{BS}(p^2, n) = \frac{1}{-K(n)^{-1} + \Sigma_\Lambda(p^2)},\ K(n) = \frac{(\sqrt{2}g)^2}{M_0^2} \sum_{k=1}^{n} \left(\frac{p^2}{M_0^2}\right)^k . \tag{9}$$

The quantity $K(n)$ is the perturbative amplitude calculated from $\mathcal{L}_{\rm le}(E_{\rm le}, N_c)$ as sum of the first n terms. Clearly, $T_{BS}(p^2, n)$ is an approximated function of $T(p^2)$ of Eq. (8). The larger n, the better is the approximation. Formally: $\lim_{n\to\infty} T_{BS}(p^2, n) = T(p^2)$.

Now, let us suppose that the low-energy Lagrangian $\mathcal{L}_{\rm le}(E_{\rm le}, N_c)$ is known, while the original Lagrangian $\mathcal{L}_{\rm toy}(E_{\max}, N_c)$ is unknown. (This is indeed the case of the real hadronic world, where only $\mathcal{L}_{\chi PT}$ or $\mathcal{L}_{\chi PT+VM}$ are known, but not $\mathcal{L}_{had}(E_{\max} \simeq 2$ GeV$, N_c)$). Moreover, we concentrate on the usually considered case in the literature: only the first term in the expansion of $\mathcal{L}_{\rm le}(E_{\rm le}, N_c)$ is kept. This means that for $n = 1$ the low-energy Lagrangian of the toy-model reads

$$\mathcal{L}_{\rm le}(E_{\rm le}, N_c) = \frac{1}{2} (\partial_\mu \varphi)^2 - \frac{1}{2} m^2 \varphi^2 + L^{(1)} \varphi^2 (-\Box) \varphi^2 , \tag{10}$$

where L^1 is now an unknown parameter. Moreover, the cutoff employed in the loop function, denoted by $\tilde{\Lambda}$, is also unknown from the perspective of the low-energy theory. The question is the following: what can we say from this point of view about the state S? Is it possible to fit the two parameters $L^{(1)}$ and $\tilde{\Lambda}$ in such a way that for $N_c = 3$ the approximated curve $T_{BS}(p^2, 1)$ reproduces the 'correct' result?

The answer is positive (see Fig. 2, first row). One can reobtain the 'bump' of the S state even in the framework of the low-energy Lagrangian $\mathcal{L}_{\rm le}$, in which the field S is not present. What we are actually doing is a reconstruction of the state S: the state S has been previously integrated out, and then it has been reobtained through a BS-unitarization study. However, it is clear that the S state, just as the previously discussed weak W boson, is not a dynamically generated molecular state of two φ fields! This is clear by the way we constructed our toy-model. However, if one would not know $\mathcal{L}_{\rm toy}$ but only the low-energy Lagrangian $\mathcal{L}_{\rm le}$, one could be led to this incorrect interpretation of the nature of the S state.

Further comments are in order:

(i) When increasing N_c, the correct T-matrix of Eq. (8) becomes narrower in agreement with the large-N_c expectations. On the contrary, the approximated form $T_{BS}(N_c, 1)$ of Eq. (9) fades out in this limit, see Fig. 2. This shows that the BS-inspired unitarization scheme does not reproduce the correct large-N_c result.

(ii) More in general, for each n the correct limit $N_c \to \infty$ cannot be reproduced. This is clear by studying the large-N_c limit of Eq. (9):

$$T_{BS}(p^2, n) \overset{N_c \to \infty}{\simeq} -K(n) . \tag{11}$$

This result follows from the fact that $K(n)$ scales as $1/N_c$ and $\Sigma_\Lambda(p^2)$ is N_c-independent. The quantity $K(n)$ is a polynomial of order n in p^2 and therefore has no pole for any finite value of p^2. This implies the incorrect result that $M_S \to \infty$ for $N_c \to \infty$.

(iii) For $n \geq 2$ it is possible to recast the BS-scheme in such a way that the correct large-N_c result is obtained [6]. However, this is not possible for the case $n = 1$, which is generally considered for explicit hadronic calculations. It is interesting to note that the IAM unitarization scheme, which is also applicable for $n \geq 2$, is in agreement with the large-N_c results. For a comparative study of different unitarization schemes see also Ref. [10].

(iv) In the examples of Fig. 2, first row, the required value of $L^{(1)}$ is for both cases $g_0 = 1.5$ GeV and $g_0 = 5$ GeV close to the correct value $\frac{g(N_c)^2}{2M_0^4}$. However, the required value of the cutoff $\tilde{\Lambda}$ varies sizably in the two cases: while for $g_0 = 5$ GeV one has $\tilde{\Lambda} = \Lambda$ (i.e., in agreement with the 'correct result') ,in the case $g_0 = 1.5$ GeV one has the unnatural value $\tilde{\Lambda} = 15000\Lambda$.

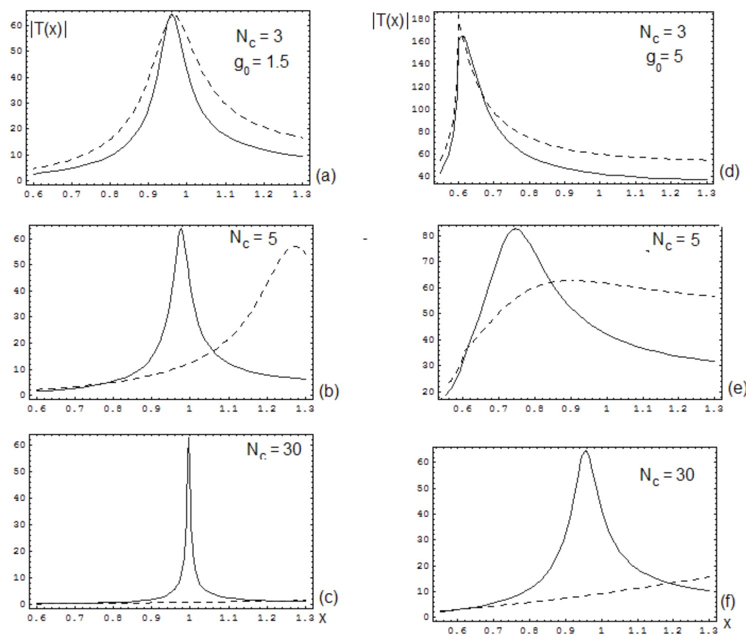

Figure 2: The solid line described the behavior of $\left|T(p^2)\right|$ of the T-matrix of Eq. (8) for $N_c = 3$, 5 and 30 and for $g_0 = 1.5$ GeV (left) and $g_0 = 5$ GeV (right) The numerical values $M_0 = 1$ GeV and $\Lambda = 1.5$ GeV have been used. The dashed line describes the approximated BS matrix $\left|T_{BS}(p^2, 1)\right|$. Although for $N_c = 3$ the two curves are similar, the behavior at large N_c is utterly different: while the solid line becomes (correctly) narrower for $N_c >> 1$, the BS-approximation shows the wrong large-N_c behavior.

(v) The cutoff $\tilde{\Lambda}$ of the low-energy version of the toy model has been taken, just as Λ_{QCD}, as large-N_c independent. This is indeed a natural choice. However, even including a direct large-N_c dependence of $\tilde{\Lambda}$, the qualitative features at large-N_c do not change. This is due to the fact that the loop function depends only logarithmically on the cutoff.

2.4 Dynamical reconstruction of the states $f_2(1270)$, $a_1(1260)$, and $f_0(1370)$

The state S is present as a fundamental, quarkonium-field in the original toy-model $\mathcal{L}_{\text{toy}}$, it is then integrated out to obtain the low-energy toy-model $\mathcal{L}_{\text{le}}$, and finally it is reobtained, i.e. *reconstructed*, via a BS-unitarization of $\mathcal{L}_{\text{le}}$. In this last step it may 'looks like' a molecular state of two φ fields, however we know that this interpretation is not correct.

The very same interpretation is now proposed for the states $f_2(1270)$, $a_1(1260)$, and $f_0(1370)$: we argue that they are quark-antiquark fields originally present in the Lagrangian $\mathcal{L}_{had}(E_{\max} \simeq 2$ GeV$, N_c = 3)$; the low-energy Lagrangian $\mathcal{L}_{\chi PT+VM}$ is (formally) calculable out of $\mathcal{L}_{had}(E_{\max} \simeq 2$ GeV$, N_c = 3)$ by integrating out the heavier fields, including $f_2(1270)$, $a_1(1260)$, and $f_0(1370)$. This step cannot be performed explicitly because $\mathcal{L}_{had}(E_{\max} \simeq 2GeV, N_c = 3)$ is not known. Finally, the states $f_2(1270)$, $a_1(1260)$, and $f_0(1370)$ are reconstructed out of $\mathcal{L}_{\chi PT+VM}$ using BS-scheme, just as we reconstructed the S field out of the low-energy Lagrangian $\mathcal{L}_{\text{le}}$ of the toy model.

From this point of view the predictions of the quark-model and the results of recent BS-studies agree with each other. It is important to remark that what is here criticized is not the result of the BS-unitarization, which is a valuable and correct analysis, but only the related interpretation of the resonances as hadronic molecular states.

3 Conclusions

Some 'dynamically generated states' exist for sure: the nuclei. They are genuine bound state of more fundamental hadrons, the protons and neutrons. The question discussed in this paper concerns the identification of molecular states beyond nuclei. To this end we concentrated on the low-energy meson spectrum and formulated the following question: are the resonances $f_2(1270)$, $a_1(1260)$, and $f_0(1370)$ dynamically generated molecular states?

Our answer is negative. With the help of a toy model we have shown that the interpretation of these states as standard quark-antiquark mesons is legitimate. In this way there is no conflict between the prediction of the quark model and the findings of BS unitarizations, which then represent two alternative ways to describe the same quark-antiquark objects. We believe that the reconciliation of the quark-model with unitarization studies solves the following puzzle: the quark-antiquark interpretation works well in the tensor and axial-vector sectors, to which $f_2(1270)$ and $a_1(1260)$ belong. If these resonances would be of different nature, that agreement would have been -rather surprisingly- accidental.

Future studies are certainly needed. The fact that the quarkonium interpretation for $f_2(1270)$, $a_1(1260)$, and $f_0(1370)$ is legitimate, in agreement with present phenomenological information and in a sense also 'desirable', does not represent a conclusive proof.

The description presented in this work is applicable with minor changes also to other recently investigated mesons between 1-2 GeV (such as the other members of the tensor and axial-vector nonets) and to dynamically generated (or reconstructed) states in the baryon and heavy quark sectors.

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